Домашна работа 2 по ДИС 1. Компютърни науки, 1 курс, 2 поток

Камен Димитров Младенов, Група 6

20 Ноември 2021

Задача 1. Дадена е редицата

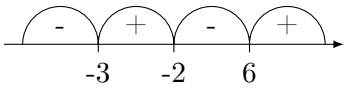
$$a_1 = \lambda,\ a_{n+1} = \frac{2a_n^2 - a_n + 6}{a_n - 6}$$

Да се изследва за сходимост спрямо стойностите на параметъра $\lambda \neq 6$.

Решение: Разглеждаме за монотонност:

$$a_{n+1} - a_n = \frac{2a_n^2 - a_n + 6}{a_n - 6} - a_n = \frac{2a_n^2 - a_n + 6 - a_n^2 + 6a_n}{a_n - 6} = \frac{a_n^2 + 5a_n + 6}{a_n - 6} = \frac{(a_n + 3)(a_n + 2)}{a_n - 6}$$

$\implies$ Корените за a_n са -3, -2 и 6



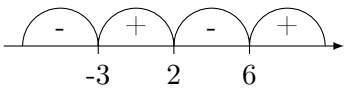
Ако съществува $l = \lim\limits_{n\to\infty} a_n$, то

$$l = \frac{2l^2 - l + 6}{l - 6} \qquad l^2 - 6l = 2l^2 - l + 6 \qquad 0 = l^2 + 5l + 6 \implies l_1 = -2\ ,\ l_2 = -3$$

Разглеждаме положението на a_{n+1} спрямо -3, -2 и 6

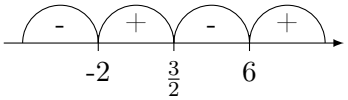
$$a_{n+1} - (-3) = \frac{2a_n^2 - a_n + 6}{a_n - 6} + 3 = \frac{2a_n^2 - a_n + 6 + 3a_n - 18}{a_n - 6} = \frac{2a_n^2 + 2a_n - 12}{a_n - 6}$$

$\implies$ Корените за a_n са -3, 2 и 6



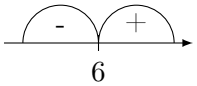
$$a_{n+1} - (-2) = \frac{2a_n^2 - a_n + 6}{a_n - 6} + 2 = \frac{2a_n^2 - a_n + 6 + 2a_n - 12}{a_n - 6} = \frac{2a_n^2 + a_n - 6}{a_n - 6}$$

$\implies$ Корените за a_n са -2, $\frac{3}{2}$ и 6



$$a_{n+1} - 6 = \frac{2a_n^2 - a_n + 6}{a_n - 6} - 6 = \frac{2a_n^2 - a_n + 6 - 6a_n + 36}{a_n - 6} = \frac{2a_n^2 - 7a_n + 42}{a_n - 6} > 0$$

$\implies$ Единствения корен за a_n е 6



Разглеждаме положението на a_{n+1} спрямо $\frac{3}{2}$ и 2

$$a_{n+1} - \frac{3}{2} = \frac{2a_n^2 - a_n + 6}{a_n - 6} - \frac{3}{2} = \frac{2a_n^2 - a_n + 6 - \frac{3}{2}a_n + 9}{a_n - 6} = \frac{2a_n^2 - \frac{5}{2}a_n + 15}{a_n - 6} > 0$$

$\implies$ Единствения корен за a_n е 6

$$a_{n+1} - 2 = \frac{2a_n^2 - a_n + 6}{a_n - 6} - 2 = \frac{2a_n^2 - a_n + 6 - 2a_n + 12}{a_n - 6} = \frac{2a_n^2 - 3a_n + 18}{a_n - 6} > 0$$

$\implies$ Единствения корен за a_n е 6

Разглеждаме положението на a_{n+1}

$$a_{n+1} = \frac{2a_n^2 - a_n + 6}{a_n - 6} > 0 \implies \text{Единствения корен за } a_n \text{ е } 6$$

a_n	$-\infty$	-3		-2		$\frac{3}{2}$		2		6	$+\infty$
$a_{n+1} - a_n$	-	$\vdots$	+	$\vdots$	-	$\vdots$	-	$\vdots$	-	$\vdots$	+
$a_{n+1} - (-3)$	-	$\vdots$	+	$\vdots$	+	$\vdots$	+	$\vdots$	-	$\vdots$	+
$a_{n+1} - (-2)$	-	$\vdots$	-	$\vdots$	+	$\vdots$	-	$\vdots$	-	$\vdots$	+
$a_{n+1} - 6$	-	$\vdots$	-	$\vdots$	-	$\vdots$	-	$\vdots$	-	$\vdots$	+
$a_{n+1} - \frac{3}{2}$	-	$\vdots$	-	$\vdots$	-	$\vdots$	-	$\vdots$	-	$\vdots$	+
$a_{n+1} - 2$	-	$\vdots$	-	$\vdots$	-	$\vdots$	-	$\vdots$	-	$\vdots$	+
a_{n+1}	-	$\vdots$	-	$\vdots$	-	$\vdots$	-	$\vdots$	-	$\vdots$	+

[TODO: FINISH]

Задача 2. Да се пресметне границата

$$\lim_{x\to\infty} \frac{\sqrt{2x^2+1} - \sqrt[3]{4x^2-1}}{\sqrt[4]{x^4+7} - \sqrt[5]{x^4-3}}$$

Решение:

$$\lim_{x\to\infty} \frac{\sqrt{2x^2+1} - \sqrt[3]{4x^2-1}}{\sqrt[4]{x^4+7} - \sqrt[5]{x^4-3}} = \lim_{x\to\infty} \frac{\cancel{x}\sqrt{2+\frac{1}{x^2}} - \cancel{x}\sqrt[3]{\frac{4}{x} - \frac{1}{x^3}}}{\cancel{x}\sqrt[4]{1+\frac{7}{x^4}} - \cancel{x}\sqrt[5]{\frac{1}{x} - \frac{3}{x^4}}} = L$$

$$\sqrt{2} \xrightarrow[x\to\infty]{} \sqrt{2}\,,\ \frac{1}{x^2} \xrightarrow[x\to\infty]{} 0\,,\ \frac{4}{x} \xrightarrow[x\to\infty]{} 0\,,\ \frac{1}{x^3} \xrightarrow[x\to\infty]{} 0\,,$$
$$\sqrt[4]{1} \xrightarrow[x\to\infty]{} \sqrt[4]{1}\,,\ \frac{7}{x^4} \xrightarrow[x\to\infty]{} 0\,,\ \frac{1}{x} \xrightarrow[x\to\infty]{} 0\,,\ \frac{3}{x^4} \xrightarrow[x\to\infty]{} 0 \qquad \implies L = \frac{\sqrt{2}}{\sqrt[4]{1}} = \sqrt{2}$$

2) $\frac{3}{2}$ и

Задача 3. Да се пресметне границата

$$\lim_{x\to 0} \frac{\sqrt[3]{5x+1} - \sqrt[4]{7x+1}}{\operatorname{arctg}(\ln(x+1))}$$

Решение:

$$\operatorname{arctg}(\ln(x+1)) \sim ln(x+1) \quad x \to 0 \qquad \sqrt[3]{5x+1} = \underbrace{(1+5x)^{\frac{1}{3}} - 1}_{\sim\frac{5x}{3} \quad x\to 0} + 1 \qquad \sqrt[4]{7x+1} = \underbrace{(1+7x)^{\frac{1}{4}} - 1}_{\sim\frac{7x}{x} \quad x\to 0} + 1$$

$$\implies \lim_{x\to 0} \frac{\sqrt[3]{5x+1} - \sqrt[4]{7x+1}}{\operatorname{arctg}(\ln(x+1))} \overset{(\sim)}{=} \lim_{x\to 0} \frac{\frac{5x}{3} + 1 - \left(\frac{7x}{4} + 1\right)}{\ln(x+1)} = \lim_{x\to 0} \frac{\frac{20x-21x}{12}}{\ln(x+1)} = \lim_{x\to 0} -\frac{x}{12\ln(x+1)} = L$$

$$\frac{x}{\ln(x+1)} \xrightarrow[x\to 0]{} 1 \implies L = -\frac{1}{12}$$

Задача 4. Да се пресметне $f^{(n)}(x)$, където

$$f(x) = \frac{x^3}{\sqrt{1-7x}}$$

Решение:

$$g(x) = \frac{1}{\sqrt{1-7x}} = (1-7x)^{\frac{1}{2}} \qquad h(x) = x^3 \qquad \Longrightarrow f(x) = g(x) \cdot h(x)$$

$$g^{(n)}(x) = \left((1-7x)^{\frac{1}{2}}\right)^{(n)} = \frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\cdots\left(\frac{1}{2}-n+1\right)\cdot x^{\frac{1}{2}-n}\cdot 1^n =$$
$$= \frac{1(-1)(-3)\cdots(1-2n+2)}{2^n}\cdot x^{\frac{1}{2}-n}$$

$$h^{(0)}(x) = h(x) = x^3 \quad h'(x) = 3x^2 \quad h''(x) = 6x \quad h'''(x) = 6 \quad h^{IV}(x) = 0 \quad \Longrightarrow h^{(n)}(x) = 0 \quad n \geq 4$$

$$f^{(n)} = \sum_{k=0}^{n} g^{(n-k)}h^{(k)} = \binom{n}{0}g^{(n)}h^{(0)} + \binom{n}{1}g^{(n-1)}h^{(1)} + \binom{n}{2}g^{(n-2)}h^{(2)} + \binom{n}{3}g^{(n-3)}h^{(3)} +$$
$$+ \underbrace{\binom{n}{4}g^{(n-4)}h^{(4)} + \cdots + \binom{n}{0}g^{(0)}h^{(n)}}_{0} =$$
$$= 1\cdot\frac{(-1)(-3)\cdots(3-2n)}{2^n}\,x^{\frac{1}{2}-n}\cdot x^3 + n\cdot\frac{(-1)(-3)\cdots(5-2n)}{2^n}\,x^{\frac{3}{2}-n}\cdot 3x^2 +$$
$$+ \frac{n!}{2(n-2)!}\cdot\frac{(-1)(-3)\cdots(7-2n)}{2^n}\,x^{\frac{5}{2}-n}\cdot 6x + \frac{n!}{6(n-3)!}\cdot\frac{(-1)(-3)\cdots(9-2n)}{2^n}\,x^{\frac{7}{2}-n}\cdot 6 =$$
$$= \frac{(-1)(-3)\cdots(3-2n)}{2^n}\,x^{\frac{7}{2}-n} + 3n\cdot\frac{(-1)(-3)\cdots(5-2n)}{2^n}\,x^{\frac{7}{2}-n} +$$
$$+ 3n(n-1)\cdot\frac{(-1)(-3)\cdots(7-2n)}{2^n}\,x^{\frac{7}{2}-n} + n(n-1)(n-2)\cdot\frac{(-1)(-3)\cdots(9-2n)}{2^n}\,x^{\frac{7}{2}-n} =$$
$$= x^{\frac{7}{2}-n}\,\frac{(-1)(-3)\cdots(9-2n)}{2^n}\left((8-2n)\cdots(3-2n) + 3n\cdot(8-2n)\cdots(5-2n) +\right.$$
$$\left. +3n^2\cdot(8-2n)(7-2n) + n^3\right)$$

Задача 5. Да се пресметне границата

$$\lim_{x\to 0}\left(\frac{\sin x}{\arcsin x}\right)^{\frac{1}{x^2}}$$

Решение: [TODO]